

### 1. Make sense of problems and persevere in solving them.

- Interpret and make meaning of the problem to find a starting point.
- Analyze what is given in order to explain to themselves the meaning of the problem.
- Plan a solution pathway instead of jumping to a solution.
- Monitor their progress and change the approach if necessary.
- See relationships between various representations.
- Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another.
- Continually ask themselves, "Does this make sense?" Can understand various approaches to solutions.

- How would you describe the problem in your own words?
- How would you describe what you are trying to find?
- What do you notice about...?
- What information is given in the problem?
- Describe the relationship between the quantities.
- Describe what you have already tried. What might you change?
- Talk me through the steps you've used to this point.
- What steps in the process are you most confident about?
- What are some other strategies you might try?
- What are some other problems that are similar to this one?
- How might you use one of your previous problems to help you begin?
- How else might you organize...represent... show...?

### 2. Reason abstractly and quantitatively.

- Make sense of quantities and their relationships.
- Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships.
- Understand the meaning of quantities and are flexible in the use of operations and their properties.
- Create a logical representation of the problem.
- Attends to the meaning of quantities, not just how to compute them.

- What do the numbers used in the problem represent?
- What is the relationship of the quantities?
- How is \_\_\_\_\_ related to \_\_\_\_\_?
- What is the relationship between \_\_\_\_\_ and \_\_\_\_\_?
- What does \_\_\_\_\_ mean to you? (e.g. symbol, quantity, diagram)
- What properties might we use to find a solution?
- How did you decide in this task that you needed to use...?
- Could we have used another operation or property to solve this task? Why or why not?

### 3. Construct viable arguments and critique the reasoning of others.

- Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments.
- Justify conclusions with mathematical ideas.
- Listen to the arguments of others and ask useful questions to determine if an argument makes sense.
- Ask clarifying questions or suggest ideas to improve/revise the argument.
- Compare two arguments and determine correct or flawed logic.

- What mathematical evidence would support your solution?
- How can we be sure that...?
- How could you prove that...?
- Will it still work if...?
- What were you considering when...?
- How did you decide to try that strategy?
- How did you test whether your approach worked?
- How did you decide what the problem was asking you to find? (What was unknown?)
- Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not?
- What is the same and what is different about...?
- How could you demonstrate a counter-example?

### 4. Model with mathematics.

- Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize).
- Apply the mathematics they know to solve everyday problems.
- Are able to simplify a complex problem and identify important quantities to look at relationships.
- Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation.
- Reflect on whether the results make sense, possibly improving/revising the model.
- Ask themselves, "How can I represent this mathematically?"

- What number model could you construct to represent the problem?
- What are some ways to represent the quantities?
- What is an equation or expression that matches the diagram, number line..., chart..., table..?
- Where did you see one of the quantities in the task in your equation or expression?
- How would it help to create a diagram, graph, table...?
- What are some ways to visually represent...?
- What formula might apply in this situation?

## 5. Use appropriate tools strategically.

- Use available tools recognizing the strengths and limitations of each.
- Use estimation and other mathematical knowledge to detect possible errors.
- Identify relevant external mathematical resources to pose and solve problems.
- Use technological tools to deepen their understanding of mathematics

- What mathematical tools could we use to visualize and represent the situation?
- What information do you have?
- What do you know that is not stated in the problem?
- What approach are you considering trying first?
- What estimate did you make for the solution?
- In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative?
- Why was it helpful to use...?
- What can using a \_\_\_\_\_ show us that \_\_\_\_\_ may not?
- In what situations might it be more informative or helpful to use...?

## 6. Attend to precision.

- Communicate precisely with others and try to use clear mathematical language when discussing their reasoning.
- Understand the meanings of symbols used in mathematics and can label quantities appropriately.
- Express numerical answers with a degree of precision appropriate for the problem context.
- Calculate efficiently and accurately.

- What mathematical terms apply in this situation?
- How did you know your solution was reasonable?
- Explain how you might show that your solution answers the problem.
- What would be a more efficient strategy?
- How are you showing the meaning of the quantities?
- What symbols or mathematical notations are important in this problem?
- What mathematical language..., definitions..., properties can you use to explain...?
- How could you test your solution to see if it answers the problem?

## 7. Look for and make use of structure.

- Apply general mathematical rules to specific situations.
- Look for the overall structure and patterns in mathematics.
- See complicated things as single objects or as being composed of several objects.

- What observations do you make about...?
- What do you notice when...?
- What parts of the problem might you eliminate..., simplify...?
- What patterns do you find in...?
- How do you know if something is a pattern?
- What ideas that we have learned before were useful in solving this problem?
- What are some other problems that are similar to this one?
- How does this relate to...?
- In what ways does this problem connect to other mathematical concepts?

## 8. Look for and express regularity in repeated reasoning.

- See repeated calculations and look for generalizations and shortcuts.
- See the overall process of the problem and still attend to the details.
- Understand the broader application of patterns and see the structure in similar situations.
- Continually evaluate the reasonableness of their intermediate results

- Explain how this strategy work in other situations
- Is this always true, sometimes true or never true?
- How would we prove that...?
- What do you notice about...?
- What is happening in this situation?
- What would happen if...?
- Is there a mathematical rule for...?
- What predictions or generalizations can this pattern support?
- What mathematical consistencies do you notice ?